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Libert, Anne-Sophie; Tsiganis, Kleomenis

Published in:
Monthly Notices of the Royal Astronomical Society

DOI:
10.1111/j.1365-2966.2009.15532.x
10.1111/j.1365-2966.2009.15532.x

Publication date:
2009

Document Version
Publisher's PDF, also known as Version of record

Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **400,** 1373–1382 (2009)

doi:10.1111/j.1365-2966.2009.15532.x

Trapping in high-order orbital resonances and inclination excitation in extrasolar systems

A.-S. Libert[1,2★] and K. Tsiganis[2★]

[1]*Department of Mathematics FUNDP, 8 Rempart de la Vierge, B-5000 Namur, Belgium*
[2]*Department of Physics, University of Thessaloniki, GR-54 124 Thessaloniki, Greece*

Accepted 2009 August 11. Received 2009 July 24; in original form 2009 June 18

ABSTRACT

Exoplanetary systems in mean motion resonance (MMR) are thought to have been captured as a result of gas-induced (Type II) orbital migration, during their early evolution phases. Using three-dimensional numerical simulations, Thommes & Lissauer showed that resonant inclination excitation can occur, for a system of two planets that evolves into a 2/1 MMR by Type II migration. In this paper, we examine whether capture in higher order resonances can also result in inclination excitation. We undertake a parametric study, varying the masses and orbital parameters of the planets, as well as the migration rate and eccentricity damping rate. We show that captures in high-order resonances (such as the 3/1, 4/1 and 5/1) are also able to produce inclination excitation. The maximal mutual inclination between the two orbital planes reaches values between 20° and 70° during a simulation, depending on the masses of the planets. Inclination excitation is observed for all configurations as long as (i) the inner planet is not very massive and (ii) at least one of the planets develops an eccentricity $e >$ 0.4. Thus, our simulations imply that inclination excitation is a common outcome, as long as eccentricity damping is not too strong. On the other hand, our results suggest that planets in the exosystems HD 60532 (3/1 MMR), HD 108874 (4/1 MMR) and HD 102272 (4/1 MMR) are most probably in coplanar orbits, since they do not meet the above two constraints. Indeed, this result was verified by a series of dedicated numerical simulations.

Key words: methods: numerical – planetary systems – planetary systems: formation – planetary systems: protoplanetary discs.

1 INTRODUCTION

At present, more than 30 extrasolar multiplanet systems have been discovered. Some of these seem to be locked in a mean motion resonance (MMR). Such systems are, for example, the two-planet systems GJ 876 (c–b), HD 82943, HD 73526 and 47 Uma, all in 2/1 MMR, HD 60532 in 3/1 MMR, and HD 102272 and HD 108874 in 4/1 MMR (see Jean Schneider's catalogue on http://exoplanet.eu). A commonly accepted scenario for the origin of these resonant configurations is disc-induced differential orbital migration of the planets, which were presumably formed farther apart, towards their parent star (Type II migration). Migration forces their orbital period ratio to change, until the commensurability is achieved. From then on, the planets migrate as a pair (or even stop migrating; see Morbidelli & Crida 2007) and the period ratio is kept fixed. Lee & Peale (2002) showed that the configuration of the GJ 876 system can be obtained by such a process and, if there is no eccentricity damping due to the disc, the growth of eccentricity can be very rapid, as migration in resonance continues.

Nelson & Papaloizou (2002) investigated the possible commensurabilities among pairs of extrasolar planets that can be reached by such a migration process, showing that 2/1 and 3/1 commensurabilities are often reached, in addition to 4/1, 5/1 and 5/2 commensurabilities, which occur less frequently. Actually, the resonant capture probability depends on the order of the resonance, the migration rate and the initial planetary eccentricities (Quillen 2006). These studies were restricted to the planar case, where both planets share the same orbital plane, a configuration probably inspired by our own planetary system. As the spatial resolution of the orbits of multiplanet exosystems is still not possible, only a few studies on the formation of exoplanetary systems have been devoted to a three-dimensional evolution so far. Concerning non-resonant systems, multiplanet scattering is the mechanism generally invoked to explain the distribution of eccentricities observed. Several studies (e.g.Marzari & Weidenschilling 2002; Chatterjee et al. 2008; Juric & Tremaine 2008; Libert & Tsiganis 2009) point out that significant values of mutual inclination of surviving two-planet systems can be achieved during this type of evolution.

★E-mail: anne-sophie.libert@fundp.ac.be (A-SL); tsiganis@astro.auth.gr (KT)

Table 1. Mean motion resonance capture for two-planet systems of different masses, initially on nearly circular orbits.

m_1/m_2	$\lvert \dot{a}_2/a_2 \rvert = 4 \times 10^{-5}\,\mathrm{yr}^{-1}$	$4 \times 10^{-6}\,\mathrm{yr}^{-1}$	$1.4 \times 10^{-6}\,\mathrm{yr}^{-1}$	$4 \times 10^{-7}\,\mathrm{yr}^{-1}$
1	2/1 $*$	2/1 $*$	2/1 $*$	3/1 $*$
2	2/1 $*$	2/1 $*$	3/1 $*$	3/1 $*$
4	2/1	3/1	3/1	3/1
0.5	2/1 $*$	2/1 $*$	3/1 $*$	3/1 $*$
0.25	2/1 $*$	2/1 $*$	3/1 $*$	3/1 $*$

Note. No eccentricity damping is considered. Symbol '$*$' indicates that the inclination-type resonance occurs as the planets migrate in resonance.

Regarding the formation of resonant non-coplanar systems, Thommes & Lissauer (2003) investigated the effects of migration in resonance on orbital inclination, starting from slightly non-coplanar orbits. Indeed, a system located just outside the 2/1 MMR can be driven by differential migration into the commensurability, which is maintained in subsequent evolution, leading to a rapid eccentricity growth. They found that, once the inner planet's eccentricity is high enough, the system enters an inclination-type resonance, which induces rapid growth of the inclination of both planets. This mechanism was only considered for a system located just outside the 2/1 MMR.

In this work, we examine whether resonant inclination excitation can also occur in systems captured in higher order resonances, whose influence on the dynamics is typically weaker. We consider a slightly non-coplanar system of two planets, initially located outside the 5/1 MMR and evolving under their mutual gravitation as well as suitably chosen drag forces that mimic the effects of Type II migration. For several mass ratios and initial eccentricities of the planets, migration rates, and eccentricity damping rates, we determine (i) the frequency of capture in different MMRs and (ii) the conditions under which resonant inclination excitation can occur.

The paper is organized as follows. In Section 2, we describe the model used in our simulations. The results of our simulations are detailed in Section 3. The effects of varying the migration rate are analysed in Section 4. In Section 5, we determine the distributions of eccentricities and inclinations achieved, during resonant migration.

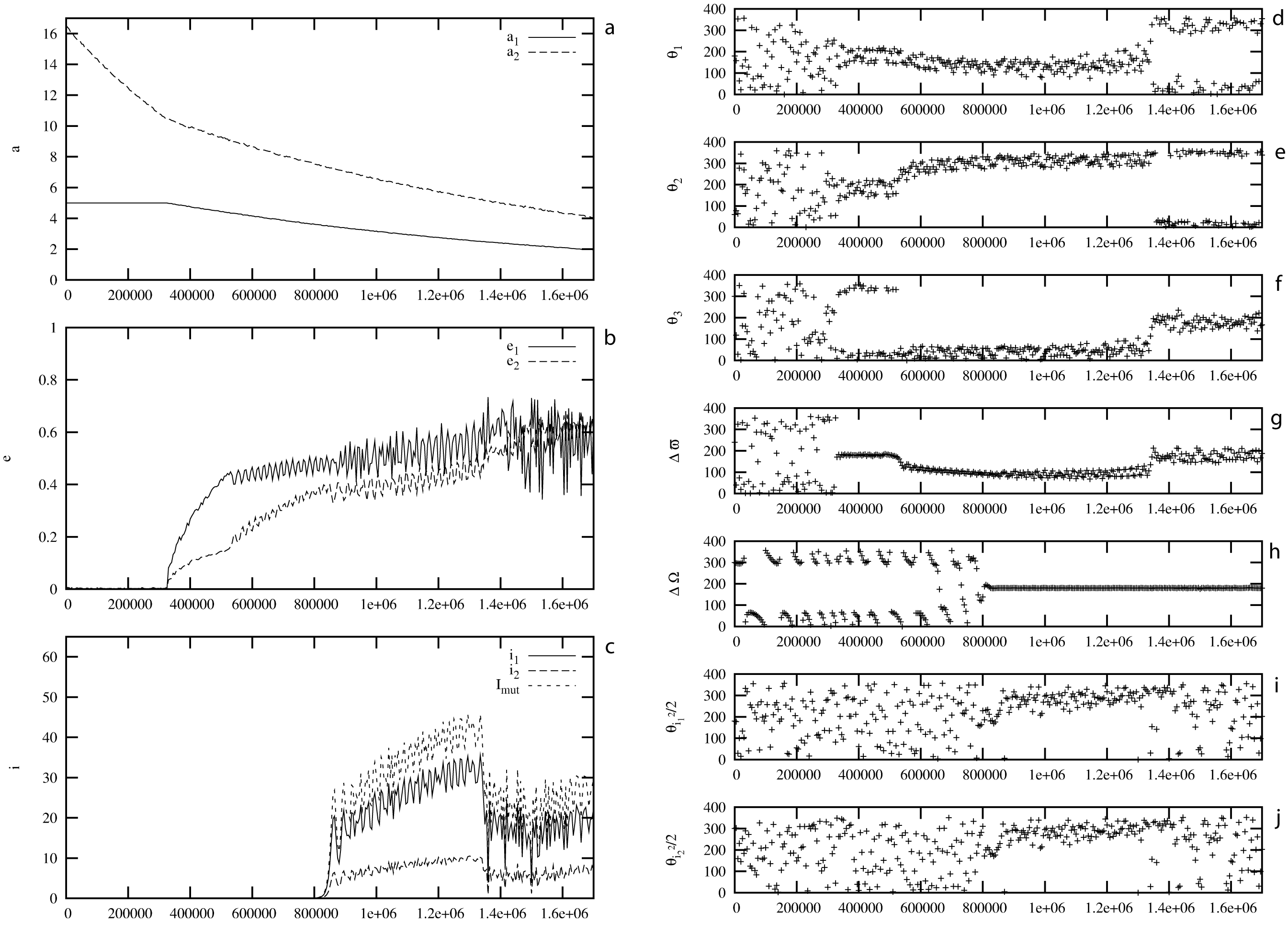

Figure 1. Inclination excitation in a 3/1 resonance capture. The migration rate is taken from equation (1): $\lvert \dot{a}_2/a_2 \rvert = 1.4 \times 10^{-6}\,\mathrm{yr}^{-1}$. The system consists of two planets of masses $m_1 = 1$ and $m_2 = 2$, on initial nearly circular orbits.

In Section 6, we discuss whether some of the detected extrasolar systems could have undergone inclination excitation during their formation. Finally, our results are summarized in Section 7.

2 NUMERICAL MODEL

A well-known scenario explaining the origin of two-planet resonant configurations assumes differential orbital migration of the planets, initially much further apart, towards their parent star. This migration process is induced by the interaction with the gas disc, in which the planets have formed. Jupiter-sized planets can carve gaps in the gaseous disc, repelling material away from their neighbourhood. The massive gaseous disc tends to refill the gaps by viscous diffusion. The balance between these processes leads to a relatively slow inward migration of the planets, whose time-scale is determined by the viscosity of the disc (Type II migration, e.g. Ward 1997). We simulate this type of planet–disc interaction using the standard recipe of applying a Stokes-type drag force (Beaugé, Michtchenko & Ferraz-Mello 2005) on the outer planet only, implicitly assuming that the inner disc is likely to be largely depleted and the dominant interactions are between the outer disc and the outer planet (see e.g. Bryden et al. 2000; Kley 2000). This force leads to an exponential decay of the outer planet's semimajor axis. However, if the planets are captured in a MMR, they subsequently migrate together towards the star, at a slower rate.

We use a Bulirsch–Stoer algorithm to integrate the three-body problem, adding the aforementioned exponential decay of the outer planet's semimajor axis. An order-of-magnitude estimate for the migration rate of a planet is given by (Ward 1997)

$$\left|\frac{\dot{a}}{a}\right| \simeq 9.4\times10^{-5}\left(\frac{\alpha}{4\times10^{-3}}\right)\left(\frac{H/a}{0.05}\right)^2 \times\left(\frac{M_0}{\mathrm{M}_\odot}\right)^{1/2}\left(\frac{a}{\mathrm{au}}\right)^{-3/2}\ \mathrm{yr}^{-1}, \tag{1}$$

where M_0 is the mass of the star, α the Shakura–Sunyaev viscosity parameter and H/a the aspect ratio of the disc. Typical values are assumed here, namely $\alpha = 4\times10^{-3}$ and $H/a = 0.05$. For our purposes, we assume a in equation (1) to be constant and equal to the initial value of semimajor axis of the outer planet's orbit. Thus, equation (1), which itself does not readily apply when two massive planets are considered (see e.g. Morbidelli & Crida 2007), is actually used here for deriving an initial estimate for the constant migration rate of the planet. However, given all uncertainties in the physical model, we decided to perform simulations with several migration rates, both smaller and larger than the one derived by equation (1).

Planet–disc interaction can also affect the eccentricity of the migrating planet (see Goldreich & Tremaine 1980; Papaloizou, Nelson & Masset 2001). In principle, eccentricity growth can also occur (see e.g. Lee & Peale 2002), although exponential damping is most commonly assumed for Jupiter-sized and smaller bodies. This effect is also reproduced by the application of the Stokes-type drag force. In the following, we adopt a damping rate of the form

$$\frac{\dot{e}}{e} = -K\left|\frac{\dot{a}}{a}\right|, \tag{2}$$

where K is arbitrarily taken equal to 0, 1, 5 or 10. Eccentricity damping is always applied on the outer planet only. Again, if the planets are captured in resonance, the response of the pair on exponential eccentricity damping is such that the eccentricities of both planets are damped.

In our simulations, we start with a system of two planets (evolving around a 1 $\mathrm{M}_\odot$ star), located at 5 and 16.5 au; the planets are outside their 5/1 MMR. For such a large value of the outer planet's semimajor axis, formula (1) gives $|\dot{a}_2/a_2| = 1.4\times10^{-6}\ \mathrm{yr}^{-1}$. As we wish to examine the effects of different migration rates on the evolution of the system we assume four different values, namely 4×10^{-5}, 4×10^{-6}, 1.4×10^{-6} and $4\times10^{-7}\ \mathrm{yr}^{-1}$. Several mass ratios and initial eccentricities of the two planets are considered in this study. In Section 3.1, we analyse the evolution of a system of planets, initially on nearly circular orbits. The effects of initially eccentric orbits and of eccentricity damping are studied in Section 3.2. The initial values of inclination for both planets are fixed to 0.01° in all our simulations. In the following section, we investigate whether capture in high-order MMRs can also result in inclination excitation.

3 RESULTS

3.1 Migration on initially circular orbits – no eccentricity damping

We consider first the evolution of a system on initial nearly circular orbits ($e_1 = e_2 = 0.001$). Moreover, eccentricity damping is considered to be ineffective, so $K = 0$. Five pairs of values are considered

Table 2. Mean motion resonance capture for two-planet systems on initially eccentric orbits.

		$\lvert\dot{a}_2/a_2\rvert = 1.4\times10^{-6}\ \mathrm{yr}^{-1}$			
m_1/m_2	ecc	$K = 0$	$K = 1$	$K = 5$	$K = 10$
1	A	2/1 *	2/1 *	2/1	2/1
1	B	3/1 *	4/1 *	3/1	3/1
1	C	2/1	3/1	3/1	3/1
1	D	3/1 *	3/1 *	3/1	3/1
1	E	3/1 *	7/2	3/1	3/1
2	A	3/1 *	3/1 *	2/1	2/1
2	B	5/2	3/1 *	3/1	3/1
2	C	5/1	4/1	5/1	2/1
2	D	3/1 *	4/1	3/1	3/1
2	E	4/1	3/1	3/1	3/1
4	A	3/1	3/1	3/1	3/1
4	B	3/1	3/1	3/1	3/1
4	C	3/1	5/1	4/1	4/1
4	D	4/1	3/1	3/1	3/1
4	E	3/1	3/1	3/1	3/1
0.5	A	3/1 *	3/1 *	2/1	2/1
0.5	B	3/1 *	3/1 *	3/1	3/1
0.5	C	5/1 *	4/1 *	5/2	2/1
0.5	D	3/1 *	3/1 *	3/1	3/1
0.5	E	3/1 *	3/1 *	3/1	3/1
0.25	A	3/1 *	3/1 *	3/1	3/1
0.25	B	4/1 *	3/1 *	3/1	3/1
0.25	C	3/1	2/1 *	3/1	3/1
0.25	D	3/1 *	3/1 *	3/1	3/1
0.25	E	3/1 *	3/1 *	3/1	3/1

Note. Symbols A,B,C,D and E denote different initial configurations: (A) $e_1 = 0.001$, $e_2 = 0.001$, (B) $e_1 = 0.1$, $e_2 = 0.001$, (C) $e_1 = 0.2$, $e_2 = 0.001$, (D) $e_1 = 0.001$, $e_2 = 0.1$ and (E) $e_1 = 0.001$, $e_2 = 0.2$. The migration rate is equal to the value given by equation (1). Different values of the eccentricity damping rate are considered. Again, symbol '*' indicates that inclination excitation occurs during the evolution.

for the masses of the planets (taking the mass of Jupiter as unit): $m_1 = m_2 = 1; m_1 = 2, m_2 = 1; m_1 = 4, m_2 = 1; m_1 = 1, m_2 = 2$ and $m_1 = 1, m_2 = 4$. Hereafter, the subscript 1 refers to the inner planet. Each pair was evolved under all four values of the migration rate (i.e. $|\dot{a}_2/a_2| = 4 \times 10^{-5}, 4 \times 10^{-6}, 1.4 \times 10^{-6}$ and 4×10^{-7} yr^{-1}).

The results of these simulations are presented in Table 1. For each mass ratio and migration rate, we describe the time evolution of the system by pointing out the MMR in which it is captured. Table 1 indicates that fast migration results in capture in 2/1 MMR. As the system remains in MMR during the subsequent migration, the eccentricities of both planets become high enough that the system enters an inclination-type resonance which induces rapid growth of the inclinations, as described by Thommes & Lissauer (2003). Inclination excitation is indicated by the presence of the symbol '$*$' in Table 1. As the migration rate decreases, planets are trapped in 3/1 MMR, instead of 2/1 MMR. This result is in agreement with the work of Quillen (2006), where it was shown that a much longer migration time-scale is required for 3/1 MMR capture.

The new result that can be found in Table 1 is that, considering orbits initially well-separated (in practice outside the 5/1 MMR), we find inclination-type resonance to occur also in a 3/1 MMR capture. Fig. 1 shows an example of this type of evolution. The outer planet migrates until both planets are captured in 3/1 MMR, which is characterized by the libration of at least one of the three resonant angles $\theta_1 = \lambda_1 - 3\lambda_2 + 2\varpi_1$, $\theta_2 = \lambda_1 - 3\lambda_2 + 2\varpi_2$ and $\theta_3 = \lambda_1 - 3\lambda_2 + \varpi_1 + \varpi_2$ (for the planar problem). In Fig. 1, the capture takes place at about 3.5×10^5 yr when all resonant angles begin to librate (around 180°, 180° and 0°, respectively, panels d, e and f). It also means that the difference of the longitudes of pericentre $\Delta\varpi = \varpi_1 - \varpi_2 = (\theta_1 - \theta_2)/2$ starts to oscillate around 0° (panel g), i.e. the planets are in apsidal alignment.

As both planets continue to migrate while in resonance, their eccentricities increase (panel b). When their values are high enough, the system enters an inclination-type resonance: the angles $\theta_{i_1^2} = 2\lambda_1 - 6\lambda_2 + 4\Omega_1$ and $\theta_{i_2^2} = 2\lambda_1 - 6\lambda_2 + 4\Omega_2$ start to librate at 8×10^5 yr (panels i, j). A rapid growth of the inclinations of both planets is observed, as well as a libration of the relative longitude of the nodes, $\Delta\Omega = \Omega_1 - \Omega_2 = (\theta_{i_1^2} - \theta_{i_2^2})/4$, around the anti-alignment state (panel h). As the amplitude of this libration is only a few degrees, the mutual inclination of the two orbital planes is approximatively the sum of their individual inclinations, and it increases very quickly to values as high as 40°, as shown by Fig. 1 (panel c). At 1.35×10^6 yr, the system leaves both inclination-type resonances, as $\theta_{i_1^2}$ and $\theta_{i_2^2}$ stop librating. Note that the system still behaves in a similar way, such that the relative longitude of the nodes $\Delta\Omega$ still librates around 180°. The resonant angles θ_1 and θ_3 switch their libration centres but the system remains stable due to the large value of mutual inclination. The inclination-growth scenario described here for the 3/1 MMR is in fact the same as the one described by Thommes & Lissauer (2003) for the 2/1 MMR.

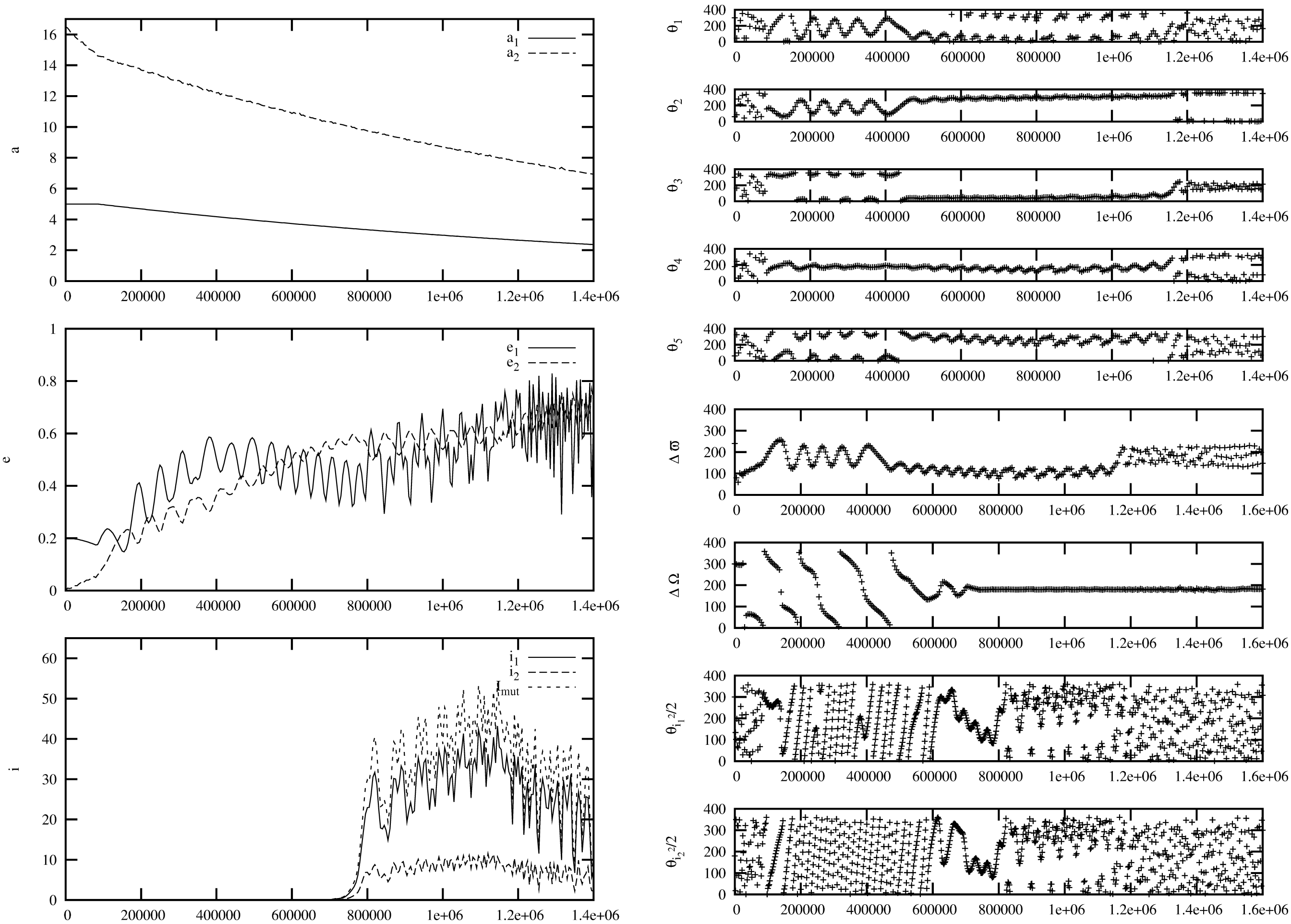

Figure 2. Inclination excitation for a 5/1 resonance capture. The migration rate is fixed to $|\dot{a}_2/a_2| = 1.4 \times 10^{-6}$ yr^{-1}. The planetary masses are $m_1 = 1$ and $m_2 = 2$ and their initial eccentricities are $e_1 = 0.2$ and $e_2 = 0.001$.

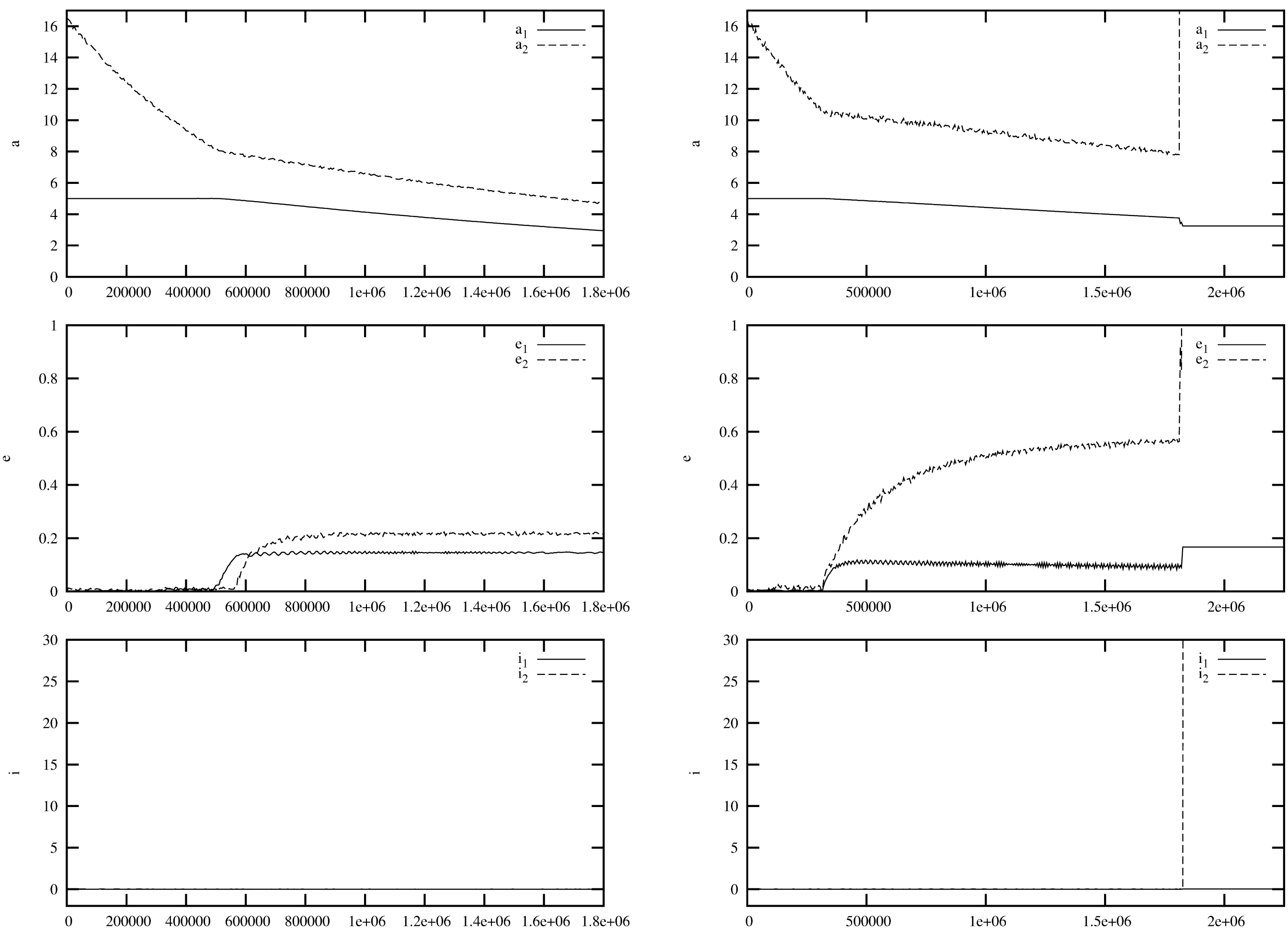

Figure 3. Left: effect of eccentricity damping of the form $\dot{e}/e = -K|\dot{a}/a|$, for $|\dot{a}_2/a_2| = 1.4 \times 10^{-6}\,\mathrm{yr}^{-1}$ and $K = 5$. The planetary masses are $m_1 = 2$ and $m_2 = 1$. Both planets start on nearly circular orbits. The growth in eccentricity is not sufficient to produce inclination excitation. Right: evolution of a system in which the mass of the inner planet is four times the mass of the outer one ($m_1 = 4$, $m_2 = 1$). The migration rate is $|\dot{a}_2/a_2| = 1.4 \times 10^{-6}\,\mathrm{yr}^{-1}$. Despite capture in the 3/1 MMR and significant eccentricity damping ($K = 1$), the system is finally destabilized because of the heavy inner planet.

The behaviour presented in Fig. 1 is a typical outcome for slow migration. Inclination excitation due to 3/1 MMR capture is produced for all mass ratios, except when the inner planet is much heavier than the outer one (see Table 1). In the case of initially circular orbits and no eccentricity damping applied, higher order resonances do not affect the behaviour of the planets. The effects of an initial non-zero eccentricity and of eccentricity damping are analysed in the following section.

3.2 Migration on initially eccentric orbits – effect of eccentricity damping

The simulations presented in Section 3.1 do not take into account that planet–disc interaction can also reduce the eccentricities of the migrating planets. In the following, we adopt damping rates corresponding to $K = 1$, 5 and 10 (see Section 2). The planets are also assumed to be initially on non-circular orbits. Five different initial states are considered: (A) $e_1 = 0.001$, $e_2 = 0.001$, (B) $e_1 = 0.1$, $e_2 = 0.001$, (C) $e_1 = 0.2$, $e_2 = 0.001$, (D) $e_1 = 0.001$, $e_2 = 0.1$ and (E) $e_1 = 0.001$, $e_2 = 0.2$. Assuming these different configurations and K values, we first perform simulations for a migration rate equal to $|\dot{a}_2/a_2| = 1.4 \times 10^{-6}\,\mathrm{yr}^{-1}$ (equation 1). Table 2 summarizes the results of these simulations.

For this value of the migration rate, the planets tend to be captured in 3/1 MMR (72 per cent of runs). About 10 per cent of the systems of Table 2 are captured in 2/1 MMR, principally those with initial nearly circular orbits. For higher initial eccentricities, captures in the 4/1, 5/1, 5/2 and even the 7/2 MMR are possible.[1] This result is in agreement with the work of Nelson & Papaloizou (2002). We also see that, for Jupiter-sized planets, the mass ratio does not influence a lot the resonance in which capture occurs. On the contrary, the strength of eccentricity damping does affect the capture: damping rates of $K = 5$ and higher favour low-order resonance captures (2/1 and 3/1 MMR).

Concerning inclination excitation, Table 2 shows that it can be produced also in the 4/1 and 5/1 MMRs. This result may seem quite unexpected, as these high-order resonances are typically considered to have a limited influence on the dynamics. Apparently, the high values of eccentricity attained, due to the on-going migration, compensate the high order of the resonant terms. An example of a 5/1 MMR capture and subsequent inclination growth is given in Fig. 2.

[1] We remind the reader that the size of a resonance increases with the eccentricity and hence even high-order resonances become of non-trivial size.

Resonant inclination excitation appears to be a common result during Type II migration, as long as eccentricity damping is not too strong. Indeed, damping rates corresponding to $K > 5$ do not allow the eccentricities to increase sufficiently for inclination-type resonance to occur, as shown by the example of Fig. 3 (left). Note also that a heavier inner planet tends to destabilize the system, before an increase in inclination can be produced – see Fig. 3 (right). For $m_1/m_2 \leq 2$ and $K \leq 1$, Table 2 shows that no less than 75 per cent of the systems go through inclination-type resonance, while remaining stable until the end of the simulation. Thus, the increase of mutual inclination seems to be a common feature in this formation scenario (smooth resonance capture by Type II migration). We found similar results by performing the same set of simulations, assuming different initial values for the orientation angles and the semimajor axis of the outer planet.

Mechanisms responsible for halting Type II migration are not well established so far. It is generally believed that the dissipation of the gas disc, sometimes after resonance capture, could prevent the planets from falling on the star. This can occur while the two planets are highly non-coplanar. Thommes & Lissauer (2003) showed that suddenly switching off the migration can produce a 2/1 resonant system with high mutual inclination. Let us consider again the example of Fig. 2. Fig. 4 illustrates the behaviour of the system formed by abruptly turning off the migration of the outer planet at 1×10^6 yr. As shown in this figure, even in this case of capture in a high-order MMR, the system can remain stable.

4 VARYING THE MIGRATION RATE

In the previous section, we considered a migration rate consistent with equation (1). However, different values of the α parameter and aspect ratio of the disc can be found in the literature. Moreover, equation (1) is only an approximation. Thus, in this section, we look for the influence of the migration rate on the results found in the previous section. We analyse the behaviour of the considered systems, evolving under larger or smaller migration rates, namely $|\dot{a}_2/a_2| = 4 \times 10^{-5}, 4 \times 10^{-6}$ and $4 \times 10^{-7}\,\mathrm{yr}^{-1}$. The results are summarized in Table 3, using the same notation as before. Only cases corresponding to $K = 0$, 1, 5 are shown, as the results for $K = 10$ are very similar to the ones for $K = 5$.

For faster migration, we find that captures in low-order resonances are favoured. This is more obvious for $|\dot{a}_2/a_2| = 4 \times 10^{-5}\,\mathrm{yr}^{-1}$, where only captures in the 2/1, 5/2 and 3/1 MMRs are observed, for the same initial orbital parameters and eccentricity damping rates as the ones of Table 2. Concerning the mutual inclinations of these systems, only a few cases of inclination excitation are observed when no eccentricity damping is considered. Indeed, the systems are generally destabilized before the inclination-type resonance can occur.

On the contrary, slow migration favours the establishment of higher order resonances. For $|\dot{a}_2/a_2| = 4 \times 10^{-7}\,\mathrm{yr}^{-1}$, nearly 30 per cent of the systems are captured in a 4/1, 5/1, 5/2 or 7/2 MMR, while almost no system is captured in a 2/1 MMR.

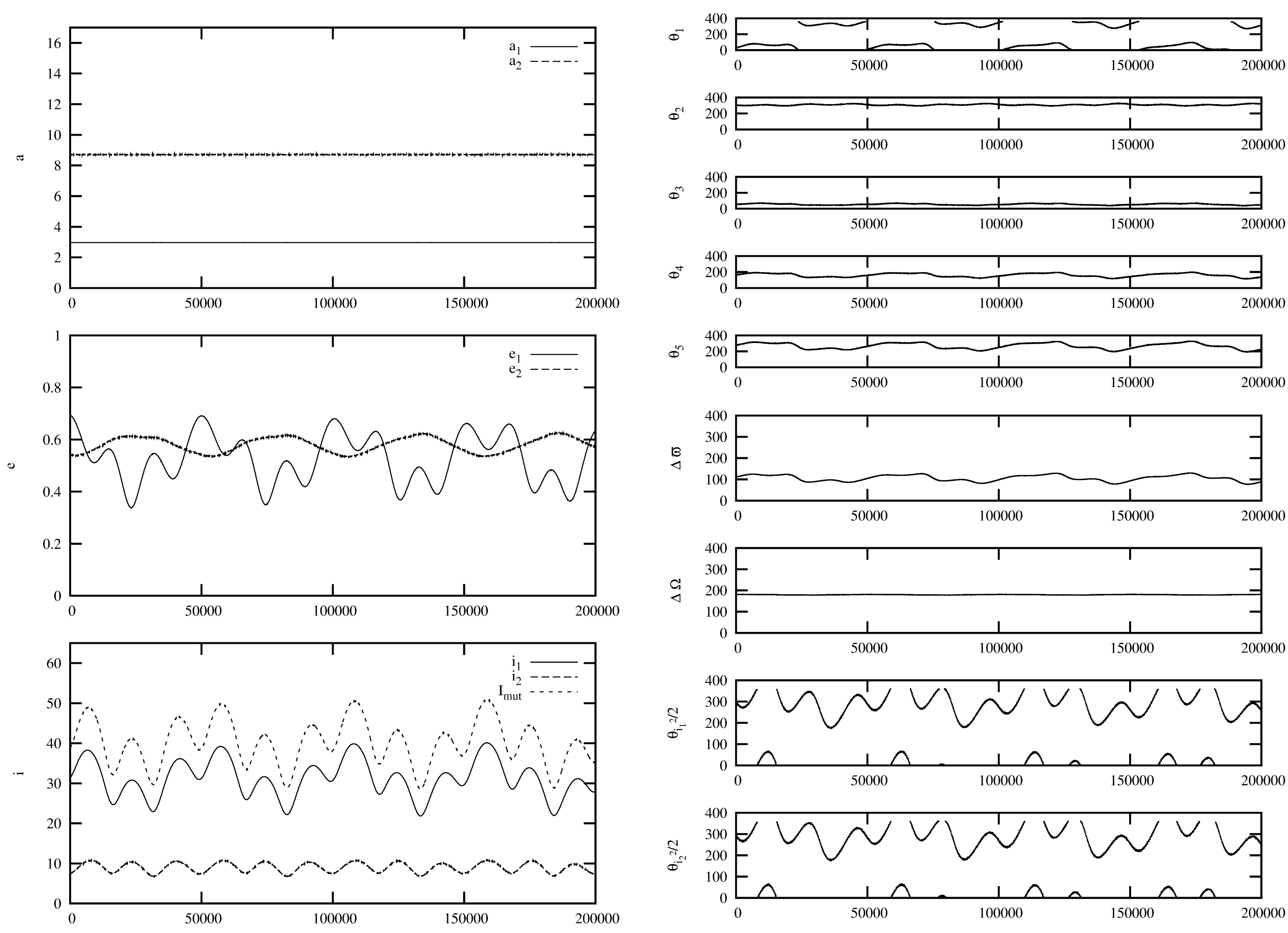

Figure 4. Evolution of the system shown in Fig. 2, after suddenly switching off the migration force (at $t = 1 \times 10^6$ yr).

However, the proportion of systems that undergo inclination-type resonance is not significantly affected. We conclude that the value of the migration rate affects strongly the resonance in which capture takes place, but only slightly the frequency of resonant inclination excitations. Thus, whatever the migration speed, the inclination excitation mechanism seems to be an important feature, if eccentricity damping is not severe ($K \leq 1$).

5 ECCENTRICITY AND INCLINATION DISTRIBUTIONS

In this section, we discuss the eccentricity and inclination distributions of the systems presented in Table 2 (for a migration rate $|\dot{a}_2/a_2| = 1.4 \times 10^{-6}\,\mathrm{yr}^{-1}$). The main observation is the diversity of configurations leading to inclination excitation.

As shown in Fig. 3 (left), the establishment of an inclination-type resonance requires that both eccentricities of these Jupiter-sized planets are high enough. In a two-planet resonant system, the eccentricity of the more massive body will not grow very much, contrary to one of the less massive body. As massive inner planets tend to destabilize the system (see Fig. 3, right), we expect less massive inner planets with higher eccentricities to be the most favourable configurations for inclination excitation. However, as shown in Table 2, for a small disparity in the masses, more massive inner planets can lead to an important increase of the outer planet's eccentricity, which also leads to inclination excitation. An example of such an evolution is given by Fig. 5 for $m_1 = 2$ and $m_2 = 1$.

To study the diversity of solutions that are compatible with the excitation of an inclination-type resonance, we plot (Fig. 6, left-hand panel) the eccentricity values of both planets, just before the inclinations start increasing. All systems of Table 2 that lead to inclination excitation are shown in this figure. We see that inclination excitation occurs for a wide range of eccentricities that depends strongly on the masses of the planets. However, no system with both eccentricities smaller than 0.4 can be found on this plot. This means that inclination-type resonance requires that at least one of the planetary eccentricities is $e \geq 0.4$. Note that the eccentricity distribution given in Fig. 6 is close to the final distribution. Indeed, when the system is in the inclination-type resonance, the eccentricities keep growing slowly, for as long as the system can remain stable.

During the inclination-type resonance, the inclination of the less massive planet grows the most. This is illustrated in Fig. 6 (right-hand panel), where the inclination values corresponding to the maximal value of the mutual inclination are given. Straight lines are level curves of constant mutual inclination, I_{mut}. As shown in the plot, the maximal I_{mut} produced by the inclination-type resonance is between 20° and 70°. Also, as m_2/m_1 increases, the maximal value of I_{mut} also increases. For $m_2/m_1 = 0.5$, I_{mut} is smaller than $\approx 35^\circ$, while for $m_2/m_1 = 4$, a value of $I_{\mathrm{mut}} > 70^\circ$ can be achieved.

6 APPLICATION TO OBSERVED EXTRASOLAR RESONANT SYSTEMS

In this section, we discuss the possibility that extrasolar systems that are found to be in a high-order MMR could have undergone inclination excitation during their formation. Indeed, resonant extrasolar systems are thought to have been formed by Type II migration, since multiplanet scattering results to only a very small

Table 3. Same results as in Table 2 for different migration rates: $|\dot{a}_2/a_2| = 4 \times 10^{-5}$, 4×10^{-6} and $4 \times 10^{-7}\,\mathrm{yr}^{-1}$.

		$|\dot{a}_2/a_2| = 4 \times 10^{-5}\,\mathrm{yr}^{-1}$			$4 \times 10^{-6}\,\mathrm{yr}^{-1}$			$4 \times 10^{-7}\,\mathrm{yr}^{-1}$		
m_1/m_2	ecc	$K = 0$	$K = 1$	$K = 5$	$K = 0$	$K = 1$	$K = 5$	$K = 0$	$K = 1$	$K = 5$
1	A	2/1 *	2/1 *	2/1	2/1 *	2/1 *	2/1	3/1 *	3/1 *	3/1
1	B	2/1	2/1	2/1	2/1 *	2/1 *	2/1	3/1 *	3/1 *	3/1
1	C	3/2	2/1	2/1	4/1 *	4/1 *	2/1	5/2	5/2	4/1
1	D	2/1	2/1 *	2/1	3/1 *	3/1 *	3/1	4/1 *	3/1 *	4/1
1	E	3/1	2/1 *	2/1	3/1	3/1 *	2/1	5/1	3/1 *	2/1
2	A	2/1 *	2/1 *	2/1	2/1 *	2/1 *	2/1	3/1 *	3/1 *	3/1
2	B	3/1	3/1 *	2/1	5/2	3/1	3/1	4/1	3/1 *	4/1
2	C	3/1	3/1	3/1	2/1	2/1 *	3/1	3/1	3/1	4/1
2	D	2/1	2/1 *	2/1	3/1 *	3/1	3/1	3/1	3/1	4/1
2	E	2/1	2/1 *	2/1	5/2	3/1	3/1	5/2	3/1	5/1
4	A	2/1	2/1 *	2/1	3/1	2/1 *	2/1	3/1	3/1	3/1
4	B	3/1	3/1	3/1	4/1	4/1	3/1	3/1	4/1	3/1
4	C	3/1	3/1	3/1	3/1	3/1	3/1	3/1	3/1	4/1
4	D	3/1	2/1	2/1	3/1	2/1	3/1	3/1	4/1	3/1
4	E	3/1	3/1	2/1	5/1	2/1	2/1	5/2	7/2	5/1
0.5	A	2/1 *	2/1 *	2/1	2/1 *	2/1 *	2/1	3/1 *	3/1 *	3/1
0.5	B	2/1	2/1 *	5/2	2/1 *	3/1	3/1	3/1 *	3/1 *	4/1
0.5	C	3/1	2/1	3/1	3/1	4/1 *	3/1 *	5/1	3/1 *	5/2
0.5	D	2/1	2/1 *	2/1	3/1 *	3/1 *	3/1	4/1 *	4/1 *	3/1
0.5	E	3/1	2/1 *	2/1	3/1	3/1 *	2/1	7/2	3/1 *	3/1
0.25	A	2/1 *	2/1 *	2/1	2/1 *	3/1 *	2/1	3/1 *	3/1 *	3/1
0.25	B	3/1	2/1 *	2/1 *	3/1 *	3/1 *	3/1	3/1 *	3/1 *	3/1
0.25	C	2/1	5/2	3/1	4/1 *	3/1 *	3/1	3/1 *	3/1 *	4/1
0.25	D	2/1	3/1 *	2/1	4/1 *	3/1 *	3/1	4/1 *	4/1 *	3/1
0.25	E	2/1	3/1	2/1	3/1 *	5/1 *	4/1	5/1 *	3/1 *	4/1

Note. The symbol '*' indicates that inclination excitation occurs during the evolution.

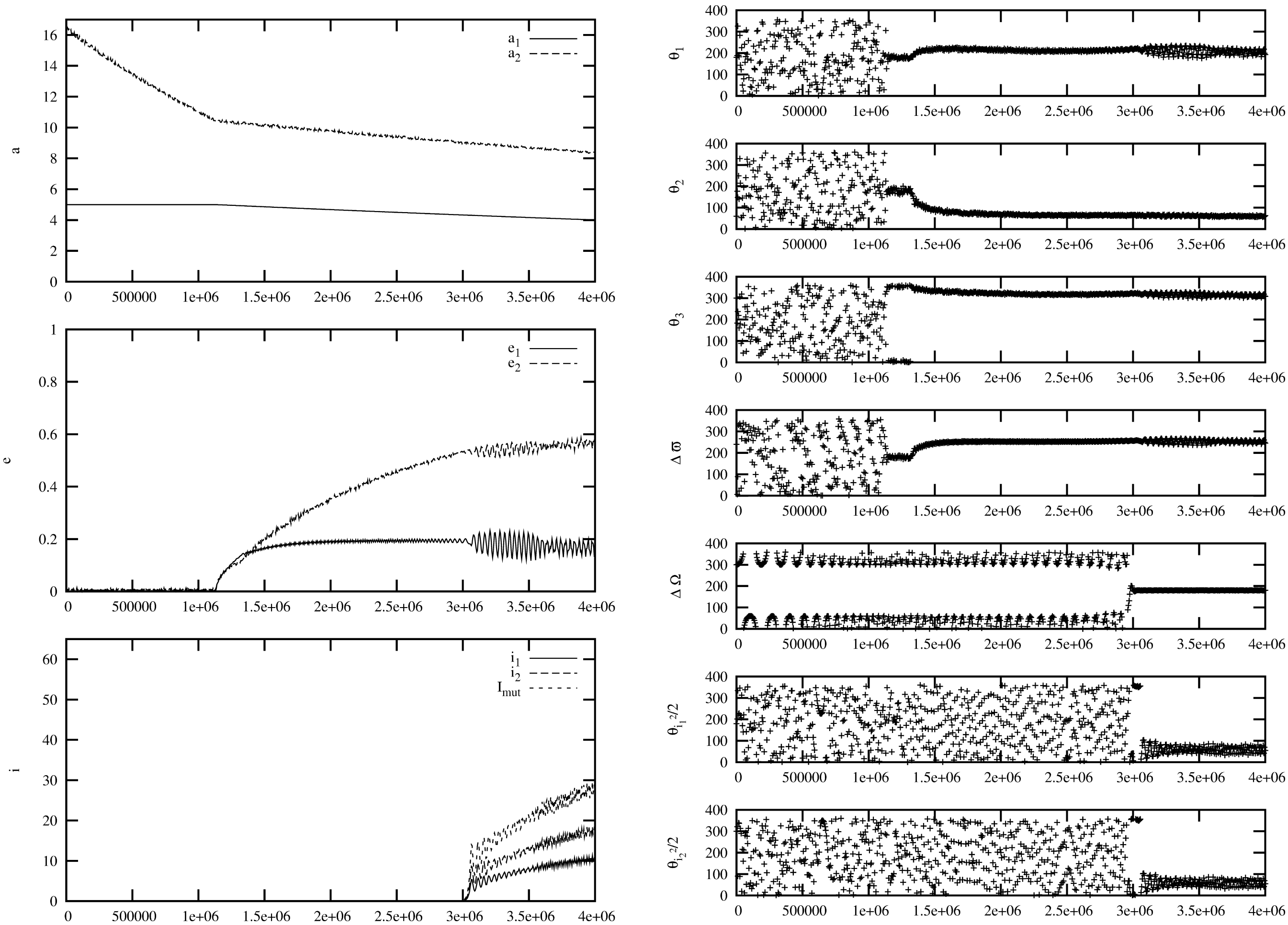

Figure 5. Evolution of a system with $m_1 = 2$ and $m_2 = 1$. The migration rate is $|\dot{a}_2/a_2| = 4 \times 10^{-7}\ \mathrm{yr}^{-1}$. No eccentricity damping is applied here.

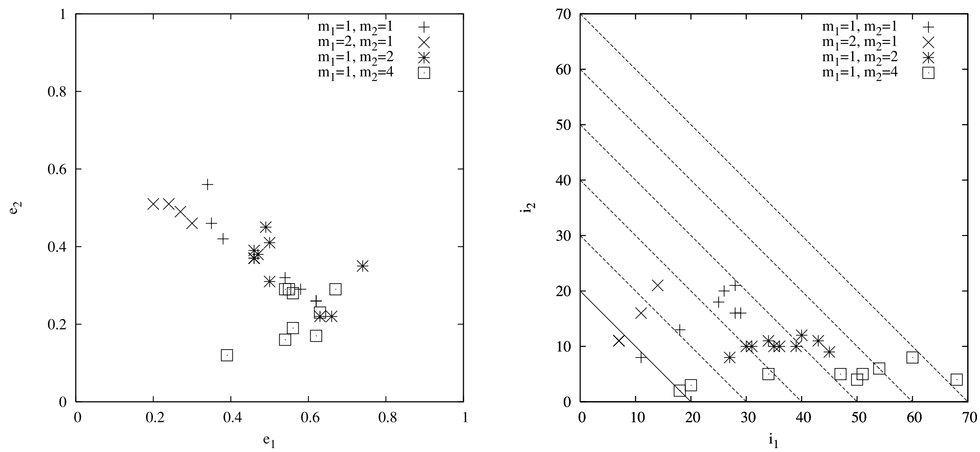

Figure 6. Left-hand panel: eccentricity distribution before inclination-type resonance for all systems of Table 2 that lead to inclination excitation (migration rate of $1.4 \times 10^{-6}\ \mathrm{yr}^{-1}$). Right-hand panel: inclination distribution corresponding to the maximal value of mutual inclination reached during inclination-type resonance. Straight lines indicate mutual inclination levels.

percentage of resonant systems (see e.g. Chatterjee et al. 2008). At least three detected two-planet systems are known to be in a high-order resonance: HD 60532, HD 108874 and HD 102272.

In Desort et al. (2008), two planetary mass companions were reported around HD 60532. The analysis of the dynamics performed by Laskar & Correia (2009) confirmed that the system is in a 3/1 MMR. Assuming coplanar motion, they found that the most stable solution is found by assuming an inclination of 20° of the system's orbital plane with respect to the plane of the sky, which corresponds to $m_1 = 3.1 M_{\mathrm{Jup}}$ and $m_2 = 7.4 M_{\mathrm{Jup}}$. Stability is also observed for inclinations in the range $i = [15°, 90°]$. For these values, the eccentricity of the inner planet oscillates roughly within

$0.1 < e_1 < 0.3$, while one of the outer planet within $0.01 < e_2 < 0.15$. We simulated Type II migration of this exosystem, in order to see whether 3/1 resonant capture is possible for mass values corresponding to inclinations in the above range, and whether inclination excitation could occur during the migration.

We performed a series of numerical simulations, similar to the ones of Section 3, for four mass values corresponding to inclinations $i = 90^\circ, 60^\circ, 40^\circ, 20^\circ$. Several migration rates [$|\dot{a}_2/a_2| = 5.1 \times 10^{-6}$, 1.7×10^{-6} – migration rate given by formula (1) – and $5.7 \times 10^{-7}\,\mathrm{yr}^{-1}$] and eccentricity damping values ($K = 0, 1, 5$) were considered for each mass configuration of the system. We find that 3/1 resonant capture is the most usual outcome and occurs, at roughly the same proportion, for all mass values considered. This formation mechanism does not argue for or against the inclination value of 20°, corresponding to the best-fitting solution of Laskar & Correia (2009). Furthermore, inclination excitation is often an outcome of the 3/1 resonant capture, leading to systems with high inclinations but also high eccentricities (generally close to $e_1 = 0.5$ and $e_2 = 0.4$). As shown by Fig. 7, stronger damping than $K = 10$ is necessary to produce the eccentricities of the HD 60532 system. However, such K values do not lead to inclination excitation. Thus, we conclude that the HD 60532 system is most likely close to coplanar.

The same is true also for the HD 108874 system, which is in 4/1 MMR (Vogt et al. 2005, see also Libert & Henrard 2007, for an analytical verification of the proximity of this system to the MMR): its eccentricities ($e_1 = 0.07$ and $e_2 = 0.25$) are too small to induce an increase in mutual inclination. Another recently discovered system believed to be in 4/1 MMR is HD 102272 (Niedzielski et al. 2009). It consists of two planets of $5.9M_{\rm Jup}$ and $2.6M_{\rm Jup}$ respectively. An interesting feature is the fact that the orbit of the outer planet is quite eccentric ($e_2 = 0.68$), while the inner one is more circular ($e_1 = 0.05$). These values are compatible with the eccentricity distribution of systems leading to inclination excitation, shown by Fig. 6. To examine the formation of this exosystem, we performed a series of simulations, for several migration rates ($|\dot{a}_2/a_2| = 5.7 \times 10^{-6}, 1.9 \times 10^{-6}$ – migration rate given by formula (1) – and $6.3 \times 10^{-7}\,\mathrm{yr}^{-1}$) and eccentricity damping values ($K = 0, 1, 5$). We find that nearly 30 per cent of the runs result in 4/1 resonant capture, leading to systems with high eccentricities when small eccentricity damping values are considered ($K = 0, 1$). However, although the mass ratio is close to 2, the inner planet is so heavy that these systems are destabilized, before an increase in inclination can be produced (see Fig. 3, right, for a similar behaviour). Thus, if confirmation on the orbital parameters is given in the following years, it seems likely that the HD 102272 system is coplanar. Note that this coplanar

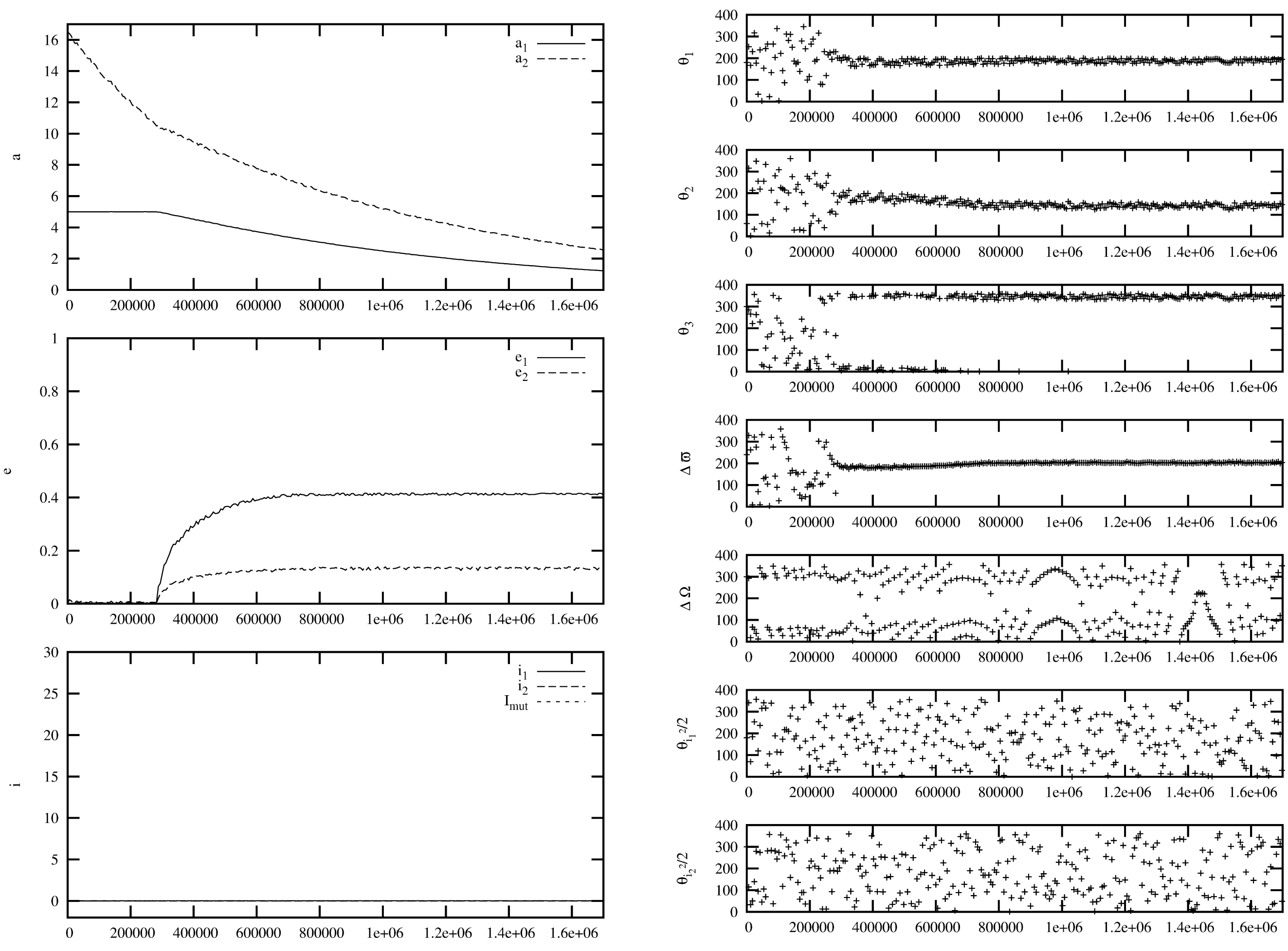

Figure 7. Evolution of a system similar to the HD 60532 system which is in a 3/1 MMR. The masses are $m_1 = 3.1\ M_{\rm Jup}$ and $m_2 = 7.3\ M_{\rm Jup}$, values corresponding to an inclination of 20° of the common orbital plane of the HD 60532 system to the plane of the sky. The migration rate is taken from equation (1): $|\dot{a}_2/a_2| = 1.7 \times 10^{-6}\,\mathrm{yr}^{-1}$. Eccentricity damping corresponding to $K = 10$ is applied on the outer body.

configuration is not easy to reach, as it might be very easily destabilized due to the heavy inner planet.

7 CONCLUSIONS

In the present work, we have studied the possibility that planetary systems, captured in high-order resonance such as the 3/1, 4/1 or 5/1 MMRs during Type II migration, undergo an inclination-type resonance, such as the one found by Thommes & Lissauer (2003) for the 2/1 MMR. This mechanism produces a rapid increase in the inclinations of the planets and could lead to the formation of resonant non-coplanar systems.

We have undertaken a parametric study, varying the masses and initial eccentricities of the planets, as well as the migration rate and eccentricity damping rate. Assuming initial nearly circular orbits and no eccentricity damping, only captures in the 2/1 and 3/1 MMR are found. For higher initial eccentricities, captures in the 4/1, 5/1, 5/2 and 7/2 MMRs are possible. Faster migration and stronger eccentricity damping are both in favour of low-order resonant captures.

The results of our simulations (summarized in Tables 2 and 3) show that inclination excitation is a common outcome in the evolution of migrating resonant systems, if eccentricity damping is not too strong. In this case, inclination excitation is observed for almost all configurations tested, except for too massive inner planets, which tend to destabilize the system. The same significant proportion of systems in inclination-type resonance is observed when varying the migration rate.

We have shown that the establishment of an inclination-type resonance requires that at least one of the eccentricities is ≥ 0.4. The maximal mutual inclination found in our simulations is between 20° and 70°, the highest values corresponding to heavier outer planets. Let us remind that the inclination-type resonance is characterized by an anti-alignment of the lines of the nodes, such that the mutual inclination is the sum of the individual inclinations.

Our study is based on modelling Type II migration by a simple exponential decay of the outer planet's semimajor axis. Of course, more accurate modelling of this process, using elaborated hydrodynamical simulations, is needed to confirm our findings. Among the limitations of our approach, we may cite the absence of modelling an inner disc and the constant migration rate (according to Kley, Peitz & Bryden 2004, the migration rate should be time-dependent to better fit hydrodynamical models). Moreover, the interaction of inclined giant planets with a gas disc has not been thoroughly studied so far and the effect of the planetary inclinations on the migration rate are not well-known.

We have discussed the possibility that some of the observed extrasolar systems in high-order MMRs might have undergone inclination excitation during their formation. The eccentricities of the HD 60532 (in 3/1 MMR) and HD 108874 (in 4/1 MMR) systems are too small to be compatible with inclination-type resonance. This was confirmed by simulations of the HD 60532 system. The HD 102272 system (in 4/1 MMR) harbour a massive inner planet which destabilizes the system before an increase in inclination is possible. Simulations of Type II migration for the HD 102272 system confirmed this result. Thus, we conclude that these three systems are most likely coplanar.

ACKNOWLEDGMENTS

The work of A-SL is supported by an FNRS post-doctoral research fellowship. A-SL thanks the people at AUTh for their hospitality during her stay in Thessaloniki.

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